

Number Play — "Figure It Out" Solutions

Complete, Section-wise Worked Solutions

Name: _____

Class: 6

Section: _____

Date: _____

This worksheet provides fully worked solutions to every "Figure It Out", "Math Talk", "Explore" and "Try This" exercise from the chapter Number Play, arranged section by section as they appear in the textbook.

Section 3.1 — Numbers Can Tell Us Things (Math Talk)

1. Can the children rearrange themselves so that the children standing at the ends say '2'?

Answer: No. A child standing at the end has only one neighbour, so they can say at most '1' (if that one neighbour is taller) — they can never say '2'.

2. Can we arrange the children in a line so that all would say only 0s?

Answer: Yes — if every child in the line is exactly the same height, no one has a taller neighbour, so everyone says '0'.

3. Can two children standing next to each other say the same number?

Answer: Yes, this is possible depending on the arrangement of heights.

4. There are 5 children of different heights. Can four of them say '1' and the last one say '0'?

Answer: Yes — if the children stand in strictly ascending (or descending) order of height, each child except the shortest has exactly one taller neighbour (saying '1'), while the shortest/tallest end child says '0'.

5. For a group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?

Answer: No. The tallest child in the group can never have a taller neighbour, so the tallest child cannot say '1'.

6. Is the sequence 0, 1, 2, 1, 0 possible?

Answer: Yes, this sequence is possible with a suitable arrangement of heights (heights rising to a peak in the middle and falling again).

7. How would you rearrange the five children so that the maximum number say '2'?

Answer: At most 2 children can say '2' in any arrangement — this can be achieved with a specific zig-zag arrangement of heights.

Section 3.2 — Supercells (Figure It Out, Page 57)

1. Colour or mark the supercells: 6828, 670, 9435, 3780, 3708, 7308, 8000, 5583, 52

Answer: Comparing each number only with its immediate left/right neighbour in the row: 6828 (>670, no left neighbour) is a supercell; 9435 (>670 and >3780) is a supercell; 7308 (>3708 and <8000, not super); 8000 (>7308 and >5583) is a supercell. Supercells: 6828, 9435, 8000.

2. Fill the table with only 4-digit numbers so the supercells are exactly the coloured cells: 5346, 1258, 9635 (pattern: coloured, not coloured, coloured)

Answer: One possible answer: 5346, 1000, 9635 — here $5346 > 1000$ and $9635 > 1000$, so 5346 and 9635 are supercells while 1000 is not. (Many other answers are possible.)

3 & 4. Fill a table with numbers 100–1000 (no repetition) to get as many supercells as possible. Out of 9 numbers, how many supercells are there?

Answer: The maximum number of supercells possible in a row of 9 numbers is 5 — achieved by placing large numbers in alternating (1st, 3rd, 5th, 7th, 9th) positions and small numbers in between.

5. Find how many supercells are possible for different numbers of cells. Is there a pattern?

Answer: For an even number of cells (2, 4, 6, ...), the maximum number of supercells is half the total: $2/2=1$, $4/2=2$, $6/2=3$, ... For an odd number of cells (1, 3, 5, 7, ...), the maximum is $(n+1)/2$: $(1+1)/2=1$, $(3+1)/2=2$, $(5+1)/2=3$, $(7+1)/2=4$, ... The strategy is to fill alternate cells with large numbers, starting from the first cell.

6. Can you fill a table without repeating numbers such that there are NO supercells?

Answer: No — the cell containing the greatest number among all the numbers chosen will always be greater than its neighbours, so it will always be a supercell, regardless of position.

7. Will the cell with the largest number always be a supercell? Can the cell with the smallest number be a supercell?

Answer: Yes, the largest number in a table is always a supercell. No, the smallest number can never be a supercell, since every one of its neighbours will be larger than it.

8. Fill a table such that the cell with the second largest number is NOT a supercell.

Answer: Sample arrangement (positions 1–9): 110, 100, 150, 130, 280, 200, 230, 210, 270 — here the second largest number sits next to the largest number, so it is not a supercell.

9. Fill a table such that the second largest number is not a supercell but the second smallest number IS a supercell. Is this possible?

Answer: Yes, it is possible. Example: arrange the numbers so the second smallest number sits between two even smaller numbers (making it a supercell), while the second largest number is placed right next to the largest number (so it is not a supercell).

10. Make other variations of this puzzle to challenge classmates.

Answer: Sample challenges: (a) Can you fill a 9-cell table so there are more than 5 supercells? (Answer: No, 5 is the maximum.) (b) Can you fill a 9-cell table with exactly 4 supercells? (Answer: Yes, this is possible with a suitable arrangement.)

Section 3.2 — Try This: 5-digit Grid (Page 58)

Complete a 4×4 grid using digits 1, 0, 6, 3, 9 (forming 5-digit numbers) so only coloured cells are supercells.

Answer: Sample completed grid: 96,310 | 96,301 | 36,109 | 39,160 // 96,103 | 13,609 | 60,319 | 19,306 // 13,906 | 10,396 | 60,193 | 60,931 // 10,369 | 10,963 | 10,936 | 69,031

The biggest number in the table is _____.

Answer: 96,310

The smallest even number in the table is _____.

Answer: 10,396

The smallest number greater than 50,000 in the table is _____.

Answer: 60,193

Section 3.3 — Patterns of Numbers on the Number Line (Figure It Out, Page 59)

Place the numbers 2180, 2754, 1500, 3600, 9950, 9590, 1050, 3050, 5030, 5300, 8400 on a number line from 1,000 to 10,000.

Answer: In increasing order along the line: 1050, 1500, 2180, 2754, 3600, 5030, 5300, 8400, 9590, 9950. (3050 was given as an example marker already shown at 2180/2754 in the textbook figure.)

Identify the numbers marked on number lines (a)-(d) and label the remaining positions, then circle the smallest and box the largest number in each sequence.

Answer: (a) Around 2010-2020, marked in steps of 5: 1990, 1995, 2000, 2005, 2010, 2015, 2020, 2025, 2030, 2035 — smallest 1990 (circle), largest 2035 (box). (b) Near 9996-9997, steps of 1: 9993 to 10,002 — smallest 9993, largest 10,002. (c) Near 15,077-15,083, steps of 1: 15,077 to 15,086 — smallest 15,077, largest 15,086. (d) Near 86,705-87,705, steps of 1,000: 83,705 to 92,705 — smallest 83,705, largest 92,705.

Section 3.4 — Playing with Digits (Page 60-61)

Find out how many numbers have two digits, three digits, four digits, and five digits.

Answer: 1-digit numbers: 9 (1-9); 2-digit numbers: 90 (10-99); 3-digit numbers: 900 (100-999); 4-digit numbers: 9,000 (1000-9999); 5-digit numbers: 90,000 (10000-99999).

Section 3.4 — Digit Sums (Figure It Out, Page 60)

1(a). Write other numbers whose digits add up to 14.

Answer: Some examples: 248, 653, 356, 815, 833, 12,335, 23,351.

1(b). What is the smallest number whose digit sum is 14?

Answer: 59 ($5 + 9 = 14$, and no smaller number has a digit sum of 14).

1(c). What is the largest 5-digit number whose digit sum is 14?

Answer: 95,000 ($9 + 5 + 0 + 0 + 0 = 14$).

1(d). How big a number can you form with digit sum 14? Can you make an even bigger one?

Answer: 95, 9005, 900005, 90000005, 9000000005, 9000000000005 ... — by inserting more zeros between the 9 and 5, you can always make an even bigger number. There is no limit.

3. Calculate the digit sums of 3-digit numbers with consecutive digits (e.g. 345). Do you see a pattern?

Answer: $123 \rightarrow 6$, $234 \rightarrow 9$, $345 \rightarrow 12$, $456 \rightarrow 15$, $567 \rightarrow 18$, $678 \rightarrow 21$, $789 \rightarrow 24$. Yes — every sum is a multiple of 3, and each sum is exactly 3 more than the previous one. This pattern continues as long as three consecutive digits exist.

Among the numbers 1-100, how many times does the digit '7' occur? Among 1-1000, how many times does it occur?

Answer: From 1 to 100, the digit 7 occurs 20 times. From 1 to 1000, the digit 7 occurs 300 times.

Section 3.5 — Palindromes (Explore & Puzzle Time, Page 61-62)

Write all possible 3-digit palindromes using a given set of digits, e.g. 1,2,3.

Answer: 111, 121, 131, 222, 212, 232, 313, 323, 333.

Will reversing and adding numbers repeatedly, starting with a 2-digit number, always give a palindrome? Explore.

Answer: Yes, it always eventually gives a palindrome. Example: $12 + 21 = 33$ (palindrome reached in 1 step). Another: $47 + 74 = 121$ (palindrome reached in 1 step). For some numbers it may take a few more rounds, but a palindrome is always reached.

Puzzle: I am a 5-digit palindrome. I am odd. My tens digit is double my units digit. My hundreds digit is double my tens digit. Who am I?

Answer: 12,421 — twelve thousand four hundred twenty-one. (Units=1, tens=2 which is double of 1, hundreds=4 which is double of 2, and being a palindrome, the thousands digit=2 and ten-thousands digit=1; the number is odd since it ends in 1.)

Section 3.6 — Reverse-and-Subtract with 3-digit Numbers (Page 63)

Carry out reverse-and-subtract repeatedly on a 3-digit number, e.g. 321. What number starts repeating?

Answer: $321 - 123 = 198 \rightarrow 981 - 189 = 792 \rightarrow 972 - 279 = 693 \rightarrow 963 - 369 = 594 \rightarrow 954 - 459 = 495 \rightarrow 954 - 459 = 495 \dots$ The number 495 starts repeating (this is the 3-digit Kaprekar-type constant). Try this process with other 3-digit numbers to verify.

Section 3.7 — Palindromic Times, Dates & Kaprekar (Figure It Out, Page 64)

Find all possible palindromic times on a 12-hour clock (like 4:44).

Answer: Some examples: 1:01, 2:02, 3:03 ... 9:09, 1:11, 2:22, 3:33, 4:44, 5:55, 10:01, 11:11, 12:21, 10:10, 12:12, and more — try listing all of them systematically.

Find some palindromic dates (DD/MM/YYYY) from the past.

Answer: Examples: 20/04/2004, 20/06/2006, 02/02/2020, and more.

Find all palindromic dates of a given special form from the past.

Answer: Examples: 01/02/2001, 02/02/2002, and more — think of others following the same digit pattern.

Will a year's calendar ever repeat exactly? Will all dates and days match another year's exactly?

Answer: Yes. A calendar repeats after 6 years if only one leap year falls in between, or after 5 years if two leap years fall in between (since day-of-week shifts depend on the number of leap years in the gap).

1. Choose 4 digits to make: (a) the difference between the largest and smallest 4-digit numbers greater than 5,085; (b) less than 5,085; (c) sum greater than 9,779; (d) sum less than 9,779. (Based on Pratibha's example using 2,3,4,7.)

Answer: Sample answers using digits 1, 3, 4, 7: (a) $7,431 - 1,347 = 6,084$ (greater than 5,085). Using digits 3, 3, 4, 7: (b) $7,433 - 3,347 = 4,086$ (less than 5,085); (c) $7,433 + 3,347 = 10,780$ (greater than 9,779). Using digits 1, 3, 4, 7: (d) $7,431 + 1,347 = 8,778$ (less than 9,779).

2. What is the sum of the smallest and largest 5-digit palindromes? What is their difference?

Answer: Smallest 5-digit palindrome = 10,001; Largest = 99,999. Sum = $10,001 + 99,999 = 1,10,000$. Difference = $99,999 - 10,001 = 89,998$.

3. The time is 10:01. How many minutes until the clock shows the next palindromic time? And the one after that?

Answer: Next palindromic time is 11:11, which is 1 hour 10 minutes = 70 minutes away. The one after that is 12:21, which is 2 hours 20 minutes = 140 minutes from 10:01.

4. How many rounds does 5,683 take to reach the Kaprekar constant?

Answer: $5,683 \rightarrow 8653 - 3568 = 5085$ (round 1) $\rightarrow 8550 - 5058 = 3492$ (round 2) $\rightarrow 9432 - 2349 = 7083$ (round 3) $\rightarrow 8730 - 3078 = 5652$ (round 4) $\rightarrow 6552 - 2556 = 3996$ (round 5) $\rightarrow 9963 - 3699 = 6264$ (round 6) $\rightarrow 6642 - 2466 = 4176$ (round 7) $\rightarrow 7641 - 1467 = 6174$ (round 8). It takes 8 rounds.

Section 3.8 — Making Thousands & Digit Combinations (Page 66)

Can we make 1,000 using the given middle numbers (multiples like 1500 and 400)? What about 14,000, 15,000 and 16,000?

Answer: No — 1,000 cannot be made this way, because the only number smaller than 1,000 available is 400, and 1,000 is not a multiple of 400. However: $14,000 = (1500 \times 8) + (400 \times 5) = 12,000 + 2,000$; $15,000 = 13,000 + (400 \times 5) = 13,000 + 2,000$; $16,000 = (1500 \times 8) + (400 \times 10) = 12,000 + 4,000$. Only 1,000 itself cannot be made this way.

Section 3.8 — Figure It Out (Page 66-69)

1. Write an example for each scenario: 5-digit + 5-digit > 90,250; 5-digit + 3-digit = 6-digit sum; 4-digit + 4-digit = 6-digit sum; 5-digit + 5-digit = 6-digit sum (1,00,000); 5-digit + 5-digit = 18,500; 5-digit - 5-digit < 56,503; 5-digit - 3-digit = 4-digit difference; 5-digit - 4-digit = 4-digit difference; 5-digit - 5-digit = 3-digit difference; 5-digit - 5-digit = 91,500.

Answer: • $45,000 + 45,400 = 90,400$ ($> 90,250$). • $99,999 + 999 = 1,00,998$ (5-digit + 3-digit = 6-digit sum). • Not possible: even the largest 4-digit + 4-digit sum, $9,999 + 9,999 = 19,998$, has only 5 digits, never 6. • $60,000 + 40,000 = 1,00,000$ (5-digit + 5-digit = 6-digit sum). • Not possible: the smallest possible 5-digit + 5-digit sum is $10,000 + 10,000 = 20,000$, which already exceeds 18,500. • $80,000 - 50,000 = 30,000$ ($< 56,503$). • $10,000 - 999 = 9,001$ (4-digit difference). • $12,000 - 2,500 = 9,500$ (4-digit difference). • $50,999 - 50,000 = 999$ (3-digit difference). • Not possible: the largest possible 5-digit - 5-digit difference is $99,999 - 10,000 = 89,999$, which is less than 91,500.

2. Always, Sometimes, or Never true? (a) 5-digit + 5-digit gives a 5-digit number; (b) 4-digit + 2-digit gives a 4-digit number; (c) 4-digit + 2-digit gives a 6-digit number; (d) 5-digit - 5-digit gives a 5-digit number; (e) 5-digit - 2-digit gives a 3-digit number.

Answer: (a) Only sometimes true — e.g. $20,000 + 80,000 = 1,00,000$, not a 5-digit number. (b) Only sometimes true — e.g. $9,999 + 99 = 10,098$, not a 4-digit number. (c) Never true — the greatest possible 4-digit + 2-digit sum is $9,999 + 99 = 10,098$, which is only 5 digits. (d) Only sometimes true — e.g. $12,000 - 10,000 = 2,000$, a 4-digit number, not always 5-digit. (e) Never true — even subtracting the greatest 2-digit number from the smallest 5-digit number gives $10,000 - 99 = 9,901$, a 4-digit number.

Section 3.10 — The Collatz Conjecture & Estimation (Page 69-71)

Make some more Collatz sequences starting from your favourite whole numbers. Do you always reach 1?

Answer: Example (a) starting at 28: 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Example (b) starting at 19: 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Yes, every sequence we try reaches 1 — even numbers are halved, and odd numbers are converted to even ($\times 3 + 1$) so they can eventually be halved down to 1.

3. Name some objects around you that are (a) a few thousand in number, and (b) more than ten thousand in number.

Answer: (a) A few thousand: car registration numbers, 4-digit PIN codes. (b) More than ten thousand: monthly salary figures (in rupees), mobile phone numbers.

3. Roshan estimates the cost of milk and 3 types of fruit for fruit custard for 5 people to be about ₹100. Do you agree?

Answer: It is possible with a small quantity of each fruit (buying just 1 of each type) but not possible if costlier fruits or larger quantities are bought — so it depends on the choice of fruits and quantity.

4. Estimate the distance between Gandhinagar (Gujarat) and Kohima (Nagaland).

Answer: Approximately 2,500 kilometres.

5. Sheetal, a Class 6 student, says she has spent around 13,000 hours in school till date. Do you agree?

Answer: No. With about 6 school hours a day and 200 working days a year: $13,000 \div (6 \times 200) = 13,000 \div 1,200 \approx 10.8$ years. But a Class 6 student has been in school for only about 8 years (Nursery to Class 6), so 13,000 hours is far too high — her claim is not reasonable.

7. Make some estimation questions to challenge your classmates!

Answer: Sample questions: How many students are there in your school? How many hours does a person sleep in their lifetime on average? (Many more such questions can be created.)

Section 3.12 — Figure It Out (Page 72, Final Exercise)

1. There is only one supercell in this grid: 16200 39344 29765 / 23609 62871 45306 / 19381 50319 38408. Exchange two digits of one number to create 4 supercells. Which digits should be swapped?

Answer: Exchanging the digits 1 and 6 in the number 62,871 (making it 12,876) creates 4 supercells. New grid: 16,200 39,344 29,765 / 23,609 12,876 45,306 / 19,381 50,319 38,408.

2. How many rounds does your year of birth take to reach the Kaprekar constant?

Answer: Example for the birth year 1980 (using 9810 as the starting 4-digit arrangement): $9810 - 1089 = 8721 \rightarrow 8721 - 1278 = 7443 \rightarrow 7443 - 3447 = 3996 \rightarrow 9963 - 3699 = 6264 \rightarrow 6642 - 2466 = 4176 \rightarrow 7641 - 1467 = 6174$. It takes 6 rounds for 1980. (Try this for your own birth year!)

3. Among 5-digit numbers between 35,000 and 75,000 with all odd digits, who is the largest, smallest, and closest to 50,000?

Answer: With repeated digits allowed: Largest = 73,999; Smallest = 35,111; Closest to 50,000 = 51,111. With all digits different (non-repeating): Largest = 73,951; Smallest = 35,179; Closest to 50,000 = 51,379.

6. Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.

Answer: $18,000 + 300 + 370 = 18,670$. (Many other combinations are possible — try finding more.)

7. Choose a number between 210 and 390 and create a number pattern (as in Section 3.9) that sums to this number.

Answer: Example — Number chosen: 250. Pattern 1 (pyramid): 25, 25 (top) / 50, 50, 50 (middle) / 25, 25 (bottom) — total = 250. Pattern 2 (equal rows): five rows of 10, 10, 10, 10, 10 (5 numbers per row $\times$ 5 rows $\times$ 10 = 250).

8. Recall the powers of 2 from Chapter 1. Why is the Collatz conjecture obviously correct for all numbers in this sequence?

Answer: Every power of 2 (2, 4, 8, 16, 32, ...) is even, so dividing by 2 repeatedly always gives the next smaller power of 2, continuing all the way down to 2, which when halved gives 1. So the Collatz process on any power of 2 is simply repeated halving straight down to 1.

9. Check if the Collatz Conjecture holds for the starting number 100.

Answer: 100, 50, 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 — yes, it reaches 1.

10. Starting with 0, players alternate adding numbers between 1 and 3. The first to reach 22 wins. What is the winning strategy?

Answer: The winning strategy is to be the first player, and to always add a number so that the total of both players' moves in each round is 4 (landing on 2, 6, 10, 14, 18, 22) — this guarantees the first player reaches 22 first.